THE RHOMBIC 132 – HEDRON

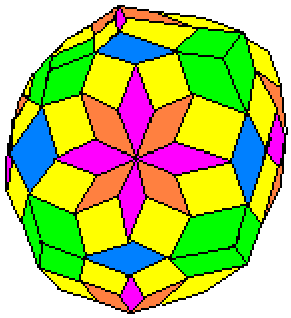

Figure 1.

This nice piece has 132 rhombic faces. There are 3 kinds of faces: the first kind is the pink and orange face, it has angle = 41.88204094 degrees = φ_1 .

$\cos\varphi_1 = \frac{7+4\sqrt{2}}{17}$. There are 48 pieces from this face.

The second kind of faces is the yellow face, it has angle = 82.66100921 degrees = φ_2 .

$\cos\varphi_2 = \frac{5-2\sqrt{2}}{17}$. There are 48 pieces from this face.

The third kind of the faces is the green and blue faces.
It has angle = 119.2776132 degrees = φ_3.

$\cos\varphi_3 = \frac{3-8\sqrt{2}}{17}$. There are 36 pieces from this face, 12 of blue, 24 of green.

The green corners make the corners of a cube, and the blue faces make the edges of the cube. The pink, orange and yellow faces make the face of the cube.

The polyhedron has $2\cdot 132 = 264$ edges, 134 corners and 132 faces.

There are 6 of 8-edged corners, $6\cdot 8 + 8 = 56$ of 3-edged corners, $6\cdot 8 = 48$ of 4-edged corners and $12\cdot 2 = 24$ of 5-edged corners.
$6 + 56 + 48 + 24 = 134$ corners.

How to construct this polyhedron?

First take a cube with edge length = 1. Draw on its face the octogons with edge length = 2h = $\sqrt{2}-1 = 0.414213562$. The length of the blue line is b.
See Fig. 1. Make the two blue lines to be parallel! See Fig. 3.

By the Fig. 4., tan 22,5 = 2h, so h = $\frac{1}{2}\tan 22{,}5° = \frac{\sqrt{2}-1}{2} = 0.207106781$.

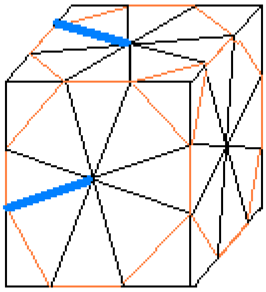

Figure 2.

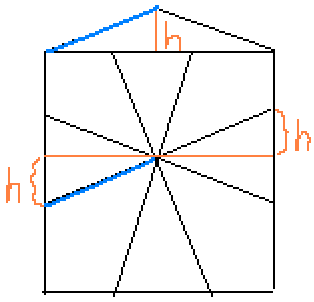

Figure 3.

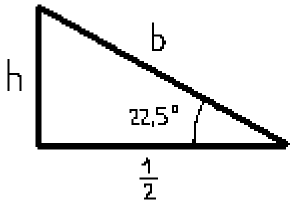

Figure 4.

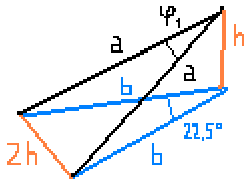

Figure 5.

$$\cos 22.5° = \frac{1}{2b} \text{ , so } b = \frac{1}{2\cos 22.5°} = 0.5411961.$$

On the Fig. 5. we can see the angle φ_1 .

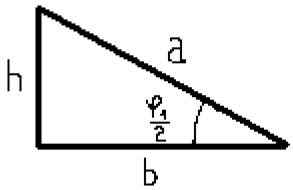

Figure 6.

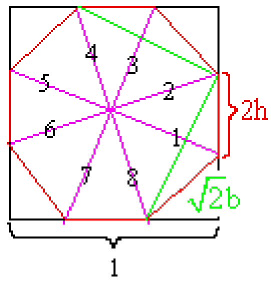

Figure 7.

By the Fig. 6. : $\tan\frac{\varphi_1}{2} = \frac{h}{b}$, and by the Fig. 4. : $\frac{h}{b} = \sin 22.5°$!

So $\varphi_1 = 2\arctan(\sin 22.5°) = 41.88204094°$

$$\cos\varphi_1 = \cos^2\frac{\varphi_1}{2} - \sin^2\frac{\varphi_1}{2} = \frac{b^2}{a^2} - \frac{h^2}{a^2} = \frac{h^2 + 0.25}{a^2} - \frac{h^2}{a^2} = \frac{0.25}{a^2}$$

$$a^2 = h^2 + b^2 = h^2 + h^2 + 0.25 = 2\left(\frac{\sqrt{2}-1}{2}\right)^2 + 0.25 = \frac{6 - 4\sqrt{2} + 1}{4} = \frac{7 - 4\sqrt{2}}{4}$$

$$\cos\varphi_1 = \frac{0.25}{a^2} = \frac{4 \cdot 0.25}{7 - 4\sqrt{2}} = \frac{1 \cdot (7 + 4\sqrt{2})}{49 - 16 \cdot 2} = \frac{7 + 4\sqrt{2}}{17}.$$

On the Fig. 7, the length of the pink lines are a.

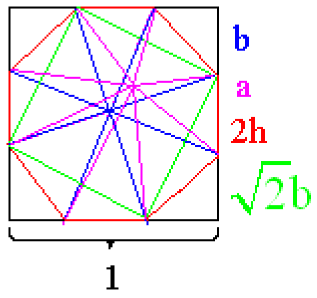

Figure 8.

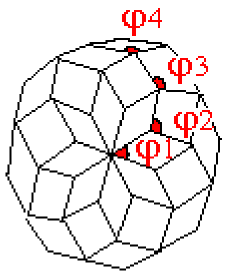

Figure 9.

The angle φ_1 is between the lines 1 and 2.
The angle φ_2 is between the lines 1 and 3.
The angle φ_3 is between the lines 1 and 4.
The angle φ_4 is between the lines 1 and 5.

$\sin\frac{\varphi_1}{2}=\frac{h}{a}$, $\sin\frac{\varphi_2}{2}=\frac{b}{\sqrt{2}a}$, $\sin\frac{\varphi_3}{2}=\frac{1}{2a}$, and $\sin\frac{\varphi_4}{2}=\frac{b}{a}=\cos\frac{\varphi_1}{2}$. So $\varphi_4 = 180 - \varphi_1$!

$$a^2=h^2+b^2=h^2+h^2+\frac{1}{4}=\frac{2(\sqrt{2}-1)^2+1}{4}=\frac{2(3-2\sqrt{2})+1}{4}=\frac{7-4\sqrt{2}}{4}$$

$$c_1=\cos\varphi_1=1-2\frac{h^2}{a^2}=\frac{a^2-2h^2}{a^2}=\frac{1}{4a^2}=\frac{1}{7-4\sqrt{2}}=\frac{7+4\sqrt{2}}{49-16\cdot 2}=\frac{7+4\sqrt{2}}{17}$$

$$c_2=\cos\varphi_2=1-2\frac{b^2}{2a^2}=\frac{a^2-b^2}{a^2}=\frac{h^2}{a^2}=\frac{3-2\sqrt{2}}{7-4\sqrt{2}}=\frac{(3-2\sqrt{2})(7+4\sqrt{2})}{17}=\frac{5-2\sqrt{2}}{17}$$

$$c_3=\cos\varphi_3=1-2\frac{1}{4a^2}=1-\frac{2}{7-4\sqrt{2}}=\frac{5-4\sqrt{2}}{7-4\sqrt{2}}=\frac{(5-4\sqrt{2})(7+4\sqrt{2})}{17}=\frac{3-8\sqrt{2}}{17}$$

$$2c_1+c_3=1$$
$$c_1+2c_2=1$$
$$c_2+s_3=1 \qquad (s_3=\sin\varphi_3=\sqrt{1-c_3^2}=\frac{12+2\sqrt{2}}{17})$$
$$c_1+c_2=s_3$$
$$c_3+4s_3=3$$
$$s_3-4c_3=2\sqrt{2}$$
$$c_2+4c_3=1-2\sqrt{2}$$

These are the properties of the Rhombic 132 – Hedron.

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