BIZONYÍTÉK AZ ÉTER LÉTÉRE?

3. RÉSZ

Vizsgáljuk most azt az általánosabb esetet, amikor $\mathrm{rot}\,\underline{\beta}$ nem nulla! Még mindig stacionáris esetet vizsgálunk, azaz $\frac{\partial}{\partial t}=0$. $\beta^2=\beta_x^2+\beta_y^2+\beta_z^2$. Most nem alkalmazunk külön feltevést, így a $\mathrm{div}\,\mathrm{grad}\frac{\beta^2}{2}=0$ feltevést sem, ezt majd utólag ellenőrizzük. (Mint tudjuk, ez a feltevés a Kerr metrika esetében is igaz.)

A Béta-metrika g_{ik} és g^{ik} tenzora most is

$$g_{ik}=\begin{pmatrix} -\left(1-\beta^2\right) & \beta_x & \beta_y & \beta_z \\ \beta_x & 1 & 0 & 0 \\ \beta_y & 0 & 1 & 0 \\ \beta_z & 0 & 0 & 1 \end{pmatrix} \text{ és } g^{ik}=\begin{pmatrix} -1 & \beta_x & \beta_y & \beta_z \\ \beta_x & 1-\beta_x^2 & -\beta_x\beta_y & -\beta_x\beta_z \\ \beta_y & -\beta_y\beta_x & 1-\beta_y^2 & -\beta_y\beta_z \\ \beta_z & -\beta_z\beta_x & -\beta_z\beta_y & 1-\beta_z^2 \end{pmatrix}$$

$$\Gamma_{kij}=\frac{1}{2}\left(\frac{\partial g_{kj}}{\partial x_i}+\frac{\partial g_{ij}}{\partial x_k}-\frac{\partial g_{ki}}{\partial x_j}\right) \text{ és } \Gamma_{ki}^{j}=g^{jm}\Gamma_{kim}$$

A következő jelöléseket alkalmazzuk:

$$B_x=\beta\partial_x\beta=\partial_x\frac{\beta^2}{2},\ B_y=\beta\partial_y\beta=\partial_y\frac{\beta^2}{2},\ B_z=\beta\partial_z\beta=\partial_z\frac{\beta^2}{2}$$

Vektoriálisan $\underline{B}=\mathrm{grad}\frac{\beta^2}{2}$

$$R_x=\frac{1}{2}\left(\partial_y\beta_z-\partial_z\beta_y\right),\ R_y=\frac{1}{2}\left(\partial_z\beta_x-\partial_x\beta_z\right),\ R_z=\frac{1}{2}\left(\partial_x\beta_y-\partial_y\beta_x\right)$$

Vektoriálisan $\underline{R} = \frac{1}{2} \text{rot}\, \underline{\beta}$.

$$D_x = \frac{1}{2}\left(\partial_y \beta_z + \partial_z \beta_y\right),\ D_y = \frac{1}{2}\left(\partial_z \beta_x + \partial_x \beta_z\right),\ D_z = \frac{1}{2}\left(\partial_x \beta_y + \partial_y \beta_x\right)$$

$$D_x = \partial_y \beta_z - R_x = \partial_z \beta_y + R_x,\ D_y = \partial_z \beta_x - R_y = \partial_x \beta_z + R_y,\ D_z = \partial_x \beta_y - R_z = \partial_y \beta_x + R_z$$

$$S_x = \beta_y R_z - \beta_z R_y,\ S_y = \beta_z R_x - \beta_x R_z,\ S_z = \beta_x R_y - \beta_y R_x$$

Vektoriálisan $\underline{S} = \underline{\beta} \times \underline{R} = \frac{1}{2} \underline{\beta} \times \text{rot}\, \underline{\beta}$.

$$A = \underline{\beta}\underline{B} = \beta_x B_x + \beta_y B_y + \beta_z B_z .$$

Most jöhetnek a Γ_{kij} tényezők:

$$\Gamma_{0ij} = \begin{pmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & R_z & -R_y \\ B_y & -R_z & 0 & R_x \\ B_z & R_y & -R_x & 0 \end{pmatrix} \qquad \Gamma_{1ij} = \begin{pmatrix} B_x & 0 & R_z & -R_y \\ \partial_x \beta_x & 0 & 0 & 0 \\ D_z & 0 & 0 & 0 \\ D_y & 0 & 0 & 0 \end{pmatrix}$$

$$\Gamma_{2ij} = \begin{pmatrix} B_y & -R_z & 0 & R_x \\ D_z & 0 & 0 & 0 \\ \partial_y \beta_y & 0 & 0 & 0 \\ D_x & 0 & 0 & 0 \end{pmatrix} \qquad \Gamma_{3ij} = \begin{pmatrix} B_z & R_y & -R_x & 0 \\ D_y & 0 & 0 & 0 \\ D_x & 0 & 0 & 0 \\ \partial_z \beta_z & 0 & 0 & 0 \end{pmatrix}$$

Ennek megfelelően a Γ_{ki}^{j} tényezők:

$$\Gamma_{0i}{}^{j} = \begin{pmatrix} -A & -B_x + \beta_x A & -B_y + \beta_y A & -B_z + \beta_z A \\ -B_x + S_x & \beta_x \left(B_x - S_x\right) & \beta_y \left(B_x - S_x\right) + R_z & \beta_z \left(B_x - S_x\right) - R_y \\ -B_y + S_y & \beta_x \left(B_y - S_y\right) - R_z & \beta_y \left(B_y - S_y\right) & \beta_z \left(B_y - S_y\right) + R_x \\ -B_z + S_z & \beta_x \left(B_z - S_z\right) + R_y & \beta_y \left(B_z - S_z\right) - R_x & \beta_z \left(B_z - S_z\right) \end{pmatrix}$$

$$\Gamma_{1i}{}^{j} = \begin{pmatrix} -B_x + S_x & \beta_x \left(B_x - S_x\right) & \beta_y \left(B_x - S_x\right) + R_z & \beta_z \left(B_x - S_x\right) - R_y \\ -\partial_x \beta_x & \beta_x \partial_x \beta_x & \beta_y \partial_x \beta_x & \beta_z \partial_x \beta_x \\ -D_z & \beta_x D_z & \beta_y D_z & \beta_z D_z \\ -D_y & \beta_x D_y & \beta_y D_y & \beta_z D_y \end{pmatrix}$$

$$\Gamma_{2i}{}^{j} = \begin{pmatrix} -B_y + S_y & \beta_x \left(B_y - S_y\right) - R_z & \beta_y \left(B_y - S_y\right) & \beta_z \left(B_y - S_y\right) + R_x \\ -D_z & \beta_x D_z & \beta_y D_z & \beta_z D_z \\ -\partial_y \beta_y & \beta_x \partial_y \beta_y & \beta_y \partial_y \beta_y & \beta_z \partial_y \beta_y \\ -D_x & \beta_x D_x & \beta_y D_x & \beta_z D_x \end{pmatrix}$$

$$\Gamma_{3i}{}^{j} = \begin{pmatrix} -B_z + S_z & \beta_x \left(B_z - S_z\right) + R_y & \beta_y \left(B_z - S_z\right) - R_x & \beta_z \left(B_z - S_z\right) \\ -D_y & \beta_x D_y & \beta_y D_y & \beta_z D_y \\ -D_x & \beta_x D_x & \beta_y D_x & \beta_z D_x \\ -\partial_z \beta_z & \beta_x \partial_z \beta_z & \beta_y \partial_z \beta_z & \beta_z \partial_z \beta_z \end{pmatrix}$$

R_{ik} kiszámolásánál még két jelölést bevezetünk:

$$L = \partial_x^2 \frac{\beta^2}{2} + \partial_y^2 \frac{\beta^2}{2} + \partial_z^2 \frac{\beta^2}{2} = \operatorname{div} \operatorname{grad} \frac{\beta^2}{2}.$$

$$T_x = \partial_y R_z - \partial_z R_y,\ T_y = \partial_z R_x - \partial_x R_z,\ T_z = \partial_x R_y - \partial_y R_x$$

Vektoriálisan $\underline{T} = \operatorname{rot} \underline{R} = \frac{1}{2} \operatorname{rot} \operatorname{rot} \underline{\beta}$.

$$R_{ik}=\partial_i\Gamma^j_{kj}-\partial_j\Gamma^j_{ik}+\Gamma^j_{im}\Gamma^m_{kj}-\Gamma^j_{mj}\Gamma^m_{ik}$$

Ebben a felírásban szerepel a Γ^j_{kj} tényező, ami egy négytagú összeg:

$$\Gamma^j_{kj}=\Gamma^0_{k0}+\Gamma^1_{k1}+\Gamma^2_{k2}+\Gamma^3_{k3}.$$

Megmutatom, hogy ez nulla, így az R_{ik} felírásában két tag mindjárt nulla lesz! Ez nagyfokú egyszerűsödést jelent.

$k=0$:

$$\Gamma^j_{0j}=\Gamma^0_{00}+\Gamma^1_{01}+\Gamma^2_{02}+\Gamma^3_{03}-A+\beta_x\left(B_x-S_x\right)+\beta_y\left(B_y-S_y\right)+\beta_z\left(B_z-S_z\right)$$

Vegyük figyelembe hogy

$A=\underline{\beta}\underline{B}=\beta_x B_x+\beta_y B_y+\beta_z B_z$, továbbá

$\underline{\beta}\underline{S}=\beta_x S_x+\beta_y S_y+\beta_z S_z=0$ mert $\underline{S}=\frac{1}{2}\underline{\beta}\times\operatorname{rot}\underline{\beta}$.

Tehát $\Gamma^j_{0j}=0$.

$k=1$:

$$\Gamma^j_{1j}=\Gamma^0_{10}+\Gamma^1_{11}+\Gamma^2_{12}+\Gamma^3_{13}=-B_x+S_x+\beta_x\partial_x\beta_x+\beta_y D_z+\beta_z D_y$$

Most

$$B_x=\partial_x\frac{\beta^2}{2}=\partial_x\frac{\beta_x^2}{2}+\partial_x\frac{\beta_y^2}{2}+\partial_x\frac{\beta_z^2}{2}=\beta_x\partial_x\beta_x+\beta_y\partial_x\beta_y+\beta_z\partial_x\beta_z$$

és

$D_z=\partial_x\beta_y-R_z$, $D_y=\partial_x\beta_z+R_y$,

valamint

$S_X=\beta_y R_z-\beta_z R_y$ figyelembevételével $\Gamma^j_{1j}=0$.

$k=2$ és $k=3$ esetében az eljárás teljesen hasonló.

Ennek megfelelően

$$R_{ik}=-\partial_j\Gamma^j_{ik}+\Gamma^j_{im}\Gamma^m_{kj}$$

$$R_{00} = -\partial_j \Gamma^j_{00} + \Gamma^j_{0m}\Gamma^m_{0j} =$$

$$= -\partial_x\left(-B_x + \beta_x A\right) - \partial_y\left(-B_y + \beta_y A\right) - \partial_z\left(-B_z + \beta_z A\right) + \Gamma^j_{0m}\Gamma^m_{0j} =$$

$$= \operatorname{div}\operatorname{grad}\frac{\beta^2}{2} - \operatorname{div}\left(\underline{\beta}\left(\underline{\beta}\operatorname{grad}\frac{\beta^2}{2}\right)\right) + \Gamma^j_{0m}\Gamma^m_{0j} = \operatorname{div}\operatorname{grad}\frac{\beta^2}{2} - \operatorname{div}\left(\underline{\beta}\left(\underline{\beta}\operatorname{grad}\frac{\beta^2}{2}\right)\right) +$$

$$+A^2 + (B_x - S_x)(B_x - \beta_x A) + (B_y - S_y)(B_y - \beta_y A) + (B_z - S_z)(B_z - \beta_z A) +$$

$$+(B_x - \beta_x A)(B_x - S_x) + \beta_x(B_x - S_x)\beta_x(B_x - S_x) +$$

$$+\left(\beta_x(B_y - S_y) - R_z\right)\left(\beta_y(B_x - S_x) + R_z\right) + \left(\beta_x(B_z - S_z) + R_y\right)\left(\beta_z(B_x - S_x) - R_y\right) +$$

$$+(B_y - \beta_y A)(B_y - S_y) + \left(\beta_x(B_y - S_y) - R_z\right)\left(\beta_y(B_x - S_x) + R_z\right) +$$

$$+\beta_y(B_y - S_y)\beta_y(B_y - S_y) + \left(\beta_y(B_z - S_z) - R_x\right)\left(\beta_z(B_y - S_y) + R_x\right) +$$

$$+(B_z - \beta_z A)(B_z - S_z) + \left(\beta_x(B_z - S_z) + R_y\right)\left(\beta_z(B_x - S_x) - R_y\right) +$$

$$+\left(\beta_y(B_z - S_z) - R_x\right)\left(\beta_z(B_y - S_y) + R_x\right) + \beta_z(B_z - S_z)\beta_z(B_z - S_z) =$$

$$= \operatorname{div}\operatorname{grad}\frac{\beta^2}{2} - \operatorname{div}\left(\underline{\beta}\left(\underline{\beta}\operatorname{grad}\frac{\beta^2}{2}\right)\right) +$$

$$+A^2 + B_xB_x - S_xB_x - B_x\beta_x A + S_x\beta_x A + B_yB_y - S_yB_y - B_y\beta_y A + S_y\beta_y A +$$

$$+ B_zB_z - S_zB_z - B_z\beta_z A + S_z\beta_z A +$$

$$+ B_xB_x - B_xS_x - \beta_x AB_x + \beta_x AS_x + \beta_x B_x\beta_x B_x - \beta_x B_x\beta_x S_x - \beta_x S_x\beta_x B_x + \beta_x S_x\beta_x S_x +$$

$$+ \beta_x B_y\beta_y B_x - \beta_x B_y\beta_y S_x + \beta_x B_yR_z - \beta_x S_y\beta_y B_x + \beta_x S_y\beta_y S_x - \beta_x S_yR_z - R_z\beta_y B_x +$$

$$+ R_z\beta_y S_x - R_zR_z +$$

$$+ \beta_x B_z\beta_z B_x - \beta_x B_z\beta_z S_x - \beta_x B_zR_y - \beta_x S_z\beta_z B_x + \beta_x S_z\beta_z S_x + \beta_x S_zR_y + R_y\beta_z B_x -$$

$$- R_y\beta_z S_x - R_yR_y +$$

$$+ B_yB_y - B_yS_y - \beta_y AB_y + \beta_y AS_y +$$

$$+ \beta_x B_y\beta_y B_x - \beta_x B_y\beta_y S_x + \beta_x B_yR_z - \beta_x S_y\beta_y B_x + \beta_x S_y\beta_y S_x - \beta_x S_yR_z - R_z\beta_y B_x +$$

$$+ R_z\beta_y S_x - R_zR_z +$$

$$+ \beta_y B_y\beta_y B_y - \beta_y B_y\beta_y S_y - \beta_y S_y\beta_y B_y + \beta_y S_y\beta_y S_y +$$

$$+ \beta_y B_z\beta_z B_y - \beta_y B_z\beta_z S_y + \beta_y B_zR_x - \beta_y S_z\beta_z B_y + \beta_y S_z\beta_z S_y - \beta_y S_zR_x - R_x\beta_z B_y +$$

$$+ R_x\beta_z S_y - R_xR_x +$$

$$+ B_zB_z - B_zS_z - \beta_z AB_z + \beta_z AS_z +$$

$+\ \beta_x B_z \beta_z B_x - \beta_x B_z \beta_z S_x - \beta_x B_z R_y - \beta_x S_z \beta_z B_x + \beta_x S_z \beta_z S_x + \beta_x S_z R_y + R_y \beta_z B_x -$

$- R_y \beta_z S_x - R_y R_y +$

$+ \beta_y B_z \beta_z B_y - \beta_y B_z \beta_z S_y + \beta_y B_z R_x - \beta_y S_z \beta_z B_y + \beta_y S_z \beta_z S_y - \beta_y S_z R_x - R_x \beta_z B_y +$

$+ R_x \beta_z S_y - R_x R_x +$

$+\ \beta_z B_z \beta_z B_z - \beta_z B_z \beta_z S_z - \beta_z S_z \beta_z B_z + \beta_z S_z \beta_z S_z =$

A színes részek az $A = \underline{\beta}\,\underline{B} = \beta_x B_x + \beta_y B_y + \beta_z B_z$,

továbbá a $\underline{\beta}\,\underline{S} = \beta_x S_x + \beta_y S_y + \beta_z S_z = 0$ figyelembevételével nullák, a maradék:

$R_{00} = \text{divgrad}\,\frac{\beta^2}{2} - \text{div}\,(\underline{\beta}\,(\underline{\beta}\,\text{grad}\,\frac{\beta^2}{2})) +$

$+ 2(B_x B_x + B_y B_y + B_z B_z + B_x S_x + B_y S_y + B_z S_z - R_x R_x - R_y R_y - R_z R_z -$

$- \beta_x S_y R_z + R_z \beta_y S_x + \beta_x S_z R_y - R_y \beta_z S_x - \beta_y S_z R_x + R_x \beta_z S_y + \beta_x B_y R_z - R_z \beta_y B_x -$

$- \beta_x B_z R_y + R_y \beta_z B_x + \beta_y B_z R_x - R_x \beta_z B_y)$

$\underline{S} = \underline{\beta} \times \underline{R}$ figyelembevételével a végeredmény: (a színes rész S^2 és $\underline{B}\,\underline{S}$)

$$R_{00} = \text{divgrad}\,\frac{\beta^2}{2} - \text{div}\,(\underline{\beta}\,(\underline{\beta}\,\text{grad}\,\frac{\beta^2}{2})) + 2\,(B^2 - R^2 + S^2 - 2\,\underline{B}\,\underline{S})$$

azaz

$$R_{00} = \text{divgrad}\,\frac{\beta^2}{2} - \text{div}\,(\underline{\beta}\,(\underline{\beta}\,\text{grad}\,\frac{\beta^2}{2})) + 2\,((\underline{B} - \underline{S})^2 - R^2).$$

Láttuk hogy a Schwarzschild és a Kerr metrika esetén $\text{divgrad}\,\frac{\beta^2}{2} = 0$ teljesül.

Itt értelemszerűen bevezettük a

$B^2 = B_x B_x + B_y B_y + B_z B_z$, az $R^2 = R_x R_x + R_y R_y + R_z R_z$

valamint az

$S^2 = S_x S_x + S_y S_y + S_z S_z$ és a $\underline{B}\,\underline{S} = B_x S_x + B_y S_y + B_z S_z$

jelöléseket.

$$R_{01} = -\partial_j \Gamma_{01}{}^j + \Gamma_{0m}{}^j \Gamma_{1j}{}^m =$$

$$= -\partial_x\left(\beta_x\left(B_x - S_x\right)\right) - \partial_y\left(\beta_y\left(B_x - S_x\right) + R_z\right) - \partial_z\left(\beta_z\left(B_x - S_x\right) - R_y\right) + \Gamma_{0m}{}^j \Gamma_{1j}{}^m =$$

$$= -\operatorname{div}(\underline{\beta}(B_x - S_x)) - T_x +$$
$$+ (B_x - S_x)A + (\partial_x \beta_x)(B_x - \beta_x A) + D_z(B_y - \beta_y A) + D_y(B_z - \beta_z A) -$$
$$- \beta_x (B_x - S_x)(B_x - S_x) + \beta_x (\partial_x \beta_x) \beta_x (B_x - S_x) +$$
$$+ \beta_x D_z(\beta_y(B_x - S_x) + R_z) + \beta_x D_y(\beta_z(B_x - S_x) - R_y) -$$
$$- (\beta_y(B_x - S_x) + R_z)(B_y - S_y) + \beta_y(\partial_x \beta_x)(\beta_x(B_y - S_y) - R_z) +$$
$$+ \beta_y D_z \beta_y (B_y - S_y) + \beta_y D_y(\beta_z(B_y - S_y) + R_x) -$$
$$- (\beta_z(B_x - S_x) - R_y)(B_z - S_z) + \beta_z(\partial_x \beta_x)(\beta_x(B_z - S_z) + R_y) +$$
$$+ \beta_z D_z(\beta_y(B_z - S_z) - R_x) + \beta_z D_y \beta_z (B_z - S_z) =$$

$$= -\operatorname{div}(\underline{\beta}(B_x - S_x)) - T_x +$$
$$+ B_x A - S_x A + (\partial_x \beta_x) B_x - (\partial_x \beta_x)\beta_x A +$$
$$+ D_z B_y - D_z \beta_y A + D_y B_z - D_y \beta_z A -$$
$$- \beta_x B_x B_x + \beta_x B_x S_x + \beta_x S_x B_x - \beta_x S_x S_x +$$
$$+ \beta_x (\partial_x \beta_x) \beta_x B_x - \beta_x (\partial_x \beta_x) \beta_x S_x +$$
$$+ \beta_x D_z \beta_y B_x - \beta_x D_z \beta_y S_x + \beta_x D_z R_z +$$
$$+ \beta_x D_y \beta_z B_x - \beta_x D_y \beta_z S_x - \beta_x D_y R_y -$$
$$- \beta_y B_x B_y + \beta_y B_x S_y + \beta_y S_x B_y - \beta_y S_x S_y - R_z B_y + R_z S_y +$$
$$+ \beta_y (\partial_x \beta_x) \beta_x B_y - \beta_y (\partial_x \beta_x) \beta_x S_y - \beta_y (\partial_x \beta_x) R_z +$$
$$+ \beta_y D_z \beta_y B_y - \beta_y D_z \beta_y S_y +$$
$$+ \beta_y D_y \beta_z B_y - \beta_y D_y \beta_z S_y + \beta_y D_y R_x -$$
$$- \beta_z B_x B_z + \beta_z B_x S_z + \beta_z S_x B_z - \beta_z S_x S_z + R_y B_z - R_y S_z +$$
$$+ \beta_z (\partial_x \beta_x) \beta_x B_z - \beta_z (\partial_x \beta_x) \beta_x S_z + \beta_z (\partial_x \beta_x) R_y +$$
$$+ \beta_z D_z \beta_y B_z - \beta_z D_z \beta_y S_z - \beta_z D_z R_x +$$
$$+ \beta_z D_y \beta_z B_z - \beta_z D_y \beta_z S_z =$$

$$= -\operatorname{div}(\underline{\beta}(B_x - S_x)) - T_x +$$
$$+ B_x A - S_x A + (\partial_x \beta_x) B_x - (\partial_x \beta_x)\beta_x A +$$

$+ (\partial_x \beta_y - R_z)B_y - (\partial_x \beta_y - R_z)\beta_y A + (\partial_x \beta_z + R_y)B_z - (\partial_x \beta_z + R_y)\beta_z A -$

$- \beta_x B_x B_x + \beta_x B_x S_x + \beta_x S_x B_x - \beta_x S_x S_x +$

$+ \beta_x (\partial_x \beta_x) \beta_x B_x - \beta_x (\partial_x \beta_x) \beta_x S_x +$

$+ \beta_x (\partial_x \beta_y - R_z) \beta_y B_x - \beta_x (\partial_x \beta_y - R_z) \beta_y S_x + \beta_x (\partial_x \beta_y - R_z) R_z +$

$+ \beta_x (\partial_x \beta_z + R_y) \beta_z B_x - \beta_x (\partial_x \beta_z + R_y) \beta_z S_x - \beta_x (\partial_x \beta_z + R_y) R_y -$

$- \beta_y B_x B_y + \beta_y B_x S_y + \beta_y S_x B_y - \beta_y S_x S_y - R_z B_y + R_z S_y +$

$+ \beta_y (\partial_x \beta_x) \beta_x B_y - \beta_y (\partial_x \beta_x) \beta_x S_y - \beta_y (\partial_x \beta_x) R_z +$

$+ \beta_y (\partial_x \beta_y - R_z) \beta_y B_y - \beta_y (\partial_x \beta_y - R_z) \beta_y S_y +$

$+ \beta_y (\partial_x \beta_z + R_y) \beta_z B_y - \beta_y (\partial_x \beta_z + R_y) \beta_z S_y + \beta_y (\partial_x \beta_z + R_y) R_x -$

$- \beta_z B_x B_z + \beta_z B_x S_z + \beta_z S_x B_z - \beta_z S_x S_z + R_y B_z - R_y S_z +$

$+ \beta_z (\partial_x \beta_x) \beta_x B_z - \beta_z (\partial_x \beta_x) \beta_x S_z + \beta_z (\partial_x \beta_x) R_y +$

$+ \beta_z (\partial_x \beta_y - R_z) \beta_y B_z - \beta_z (\partial_x \beta_y - R_z) \beta_y S_z - \beta_z (\partial_x \beta_y - R_z) R_x +$

$+ \beta_z (\partial_x \beta_z + R_y) \beta_z B_z - \beta_z (\partial_x \beta_z + R_y) \beta_z S_z.$

A színessel kiemeltek nullák, az A, az S_x és a B_x definícióját felhasználva. A maradék rész így alakul:

$R_{01} = - \operatorname{div} (\underline{\beta} (B_x - S_x)) - T_x +$

$+(\partial_x \beta_x)B_x + (\partial_y \beta_x + R_z)B_y + (\partial_z \beta_x - R_y) B_z - \beta_x S_x S_x +$

$+ \beta_x R_z \beta_y S_x + \beta_x (\partial_y \beta_x + R_z) R_z - \beta_x R_y \beta_z S_x - \beta_x (\partial_z \beta_x - R_y) R_y -$

$- \beta_y S_x S_y - R_z B_y + R_z S_y - \beta_y (\partial_x \beta_x) R_z + \beta_y R_z \beta_y S_y -$

$- \beta_y R_y \beta_z S_y + \beta_y (\partial_z \beta_x - R_y) R_x - \beta_z S_x S_z + R_y B_z - R_y S_z +$

$+ \beta_z (\partial_x \beta_x) R_y + \beta_z R_z \beta_y S_z - \beta_z (\partial_y \beta_x + R_z) R_x - \beta_z R_y \beta_z S_z$

A pirossal kiemelt részben átalakítást végeztünk a D_z és a D_y kétféle felírása szerint.

$D_z = (\partial_x \beta_y - R_z) = (\partial_y \beta_x + R_z)$ és $D_y = (\partial_x \beta_z + R_y) = (\partial_z \beta_x - R_y)$

$R_{01} = - \operatorname{div} (\underline{\beta} (B_x - S_x)) - T_x +$

$+(\partial_x \beta_x)B_x + (\partial_y \beta_x + R_z)B_y + (\partial_z \beta_x - R_y) B_z - \beta_x S_x S_x +$

$+\ \beta_x R_z \beta_y S_x + \beta_x (\partial_y \beta_x + R_z) R_z - \beta_x R_y \beta_z S_x - \beta_x (\partial_z \beta_x - R_y) R_y -$

$-\ \beta_y S_x S_y - R_z B_y + R_z S_y - \beta_y (\partial_x \beta_x) R_z + \beta_y R_z \beta_y S_y -$

$-\ \beta_y R_y \beta_z S_y + \beta_y (\partial_z \beta_x - R_y) R_x - \beta_z S_x S_z + R_y B_z - R_y S_z +$

$+\ \beta_z (\partial_x \beta_x) R_y + \beta_z R_z \beta_y S_z - \beta_z (\partial_y \beta_x + R_z) R_x - \beta_z R_y \beta_z S_z \ =$

= A piros rész $\underline{B}$ grad β_x, a kék rész $-\underline{S}$ grad β_x.

A zöldek, a narancssárgák és a barnák nullák.

Marad tehát végül:

$$R_{01} = -\,\mathrm{div}\,(\underline{\beta}\,(B_x - S_x)) - T_x \ + (\underline{B} - \underline{S})\ \mathrm{grad}\ \beta_x$$

Szimmetriaokokból

$$R_{02} = -\,\mathrm{div}\,(\underline{\beta}\,(B_y - S_y)) - T_y \ + (\underline{B} - \underline{S})\ \mathrm{grad}\ \beta_y$$

$$R_{03} = -\,\mathrm{div}\,(\underline{\beta}\,(B_z - S_z)) - T_z \ + (\underline{B} - \underline{S})\ \mathrm{grad}\ \beta_z$$

A három egyenlet egy képletbe is összefoglalható, ha figyelembevesszük hogy

$$\mathrm{div}\,(\underline{\beta}\,(B_x - S_x)) = (B_x - S_x)\ \mathrm{div}\ \underline{\beta} \ + \underline{\beta}\ \mathrm{grad}\ (B_x - S_x).$$

Ekkor

$$R_{01} = -\,(B_x - S_x)\ \mathrm{div}\ \underline{\beta} \ - \underline{\beta}\ \mathrm{grad}\ (B_x - S_x) - T_x \ + (\underline{B} - \underline{S})\ \mathrm{grad}\ \beta_x.$$

Ekkor a következő vektoranalitikai összefüggésre ismerhetünk rá:

$$\mathrm{rot}\ (\underline{\beta} \times (\underline{B} - \underline{S})) = ((\underline{B} - \underline{S}),\ \mathrm{grad})\ \underline{\beta} - (\underline{\beta},\ \mathrm{grad})(B_x - S_x) - (\underline{B} - \underline{S})\ \mathrm{div}\ \underline{\beta} +$$
$$+\ \underline{\beta}\ \mathrm{div}\ (\underline{B} - \underline{S}).$$

Ám ez az utóbbi tag éppen hiányzik. Így tehát

$$\underline{R}_0 = \mathrm{rot}\ (\underline{\beta} \times (\underline{B} - \underline{S})) - \underline{\beta}\ \mathrm{div}\ (\underline{B} - \underline{S}) - \underline{T}.$$

$\underline{R}_0$ a vektorrá összefogott R_{01}, R_{02}, R_{03} -t jelenti.

R_{11} következik:

$$R_{11} = -\,\partial_j \Gamma_{11}{}^{j} + \Gamma_{1m}{}^{j}\,\Gamma_{1j}{}^{m} =$$

$$= -\partial_x\left(\beta_x\partial_x\beta_x\right) - \partial_y\left(\beta_y\partial_x\beta_x\right) - \partial_z\left(\beta_z\partial_x\beta_x\right) + \Gamma_{1m}{}^{j}\,\Gamma_{1j}{}^{m} =$$

$$= -\operatorname{div}(\underline{\beta}\,\partial_x\beta_x) +$$

$$+ (B_x - S_x)(B_x - S_x) - (\partial_x\beta_x)\beta_x(B_x - S_x) - D_z(\beta_y(B_x - S_x) + R_z)$$

$$- D_y(\beta_z(B_x - S_x) - R_y) -$$

$$- (\partial_x\beta_x)\beta_x(B_x - S_x) + \beta_x(\partial_x\beta_x)\beta_x(\partial_x\beta_x) + \beta_y(\partial_x\beta_x)\beta_x D_z + \beta_z(\partial_x\beta_x)\beta_x D_y -$$

$$- D_z(\beta_y(B_x - S_x) + R_z) + \beta_x D_z\beta_y(\partial_x\beta_x) + \beta_y D_z\beta_y D_z + \beta_z D_z\beta_y D_y -$$

$$- D_y(\beta_z(B_x - S_x) - R_y) + \beta_x D_y\beta_z(\partial_x\beta_x) + \beta_y D_y\beta_z D_z + \beta_z D_y\beta_z D_y =$$

$$= -\operatorname{div}(\underline{\beta}\,\partial_x\beta_x) +$$

$$+ B_x B_x - B_x S_x - S_x B_x + S_x S_x - (\partial_x\beta_x)\beta_x B_x + (\partial_x\beta_x)\beta_x S_x - D_z\beta_y B_x + D_z\beta_y S_x -$$

$$- D_z R_z - D_y\beta_z B_x + D_y\beta_z S_x + D_y R_y -$$

$$- (\partial_x\beta_x)\beta_x B_x + (\partial_x\beta_x)\beta_x S_x + \beta_x(\partial_x\beta_x)\beta_x(\partial_x\beta_x) + \beta_y(\partial_x\beta_x)\beta_x D_z +$$

$$+ \beta_z(\partial_x\beta_x)\beta_x D_y -$$

$$- D_z\beta_y B_x + D_z\beta_y S_x - D_z R_z + \beta_x D_z\beta_y(\partial_x\beta_x) + \beta_y D_z\beta_y D_z + \beta_z D_z\beta_y D_y -$$

$$- D_y\beta_z B_x + D_y\beta_z S_x + D_y R_y + \beta_x D_y\beta_z(\partial_x\beta_x) + \beta_y D_y\beta_z D_z + \beta_z D_y\beta_z D_y =$$

$$= -\operatorname{div}(\underline{\beta}\,\partial_x\beta_x) +$$

$$+ B_x B_x - B_x S_x - S_x B_x + S_x S_x - (\partial_x\beta_x)\beta_x B_x + (\partial_x\beta_x)\beta_x S_x -$$

$$- (\partial_x\beta_y)\beta_y B_x + R_z\beta_y B_x + (\partial_x\beta_y)\beta_y S_x - R_z\beta_y S_x -$$

$$- (\partial_x\beta_y)R_z + R_z R_z - (\partial_x\beta_z)\beta_z B_x - R_y\beta_z B_x + (\partial_x\beta_z)\beta_z S_x + R_y\beta_z S_x +$$

$$+ (\partial_x\beta_z)R_y + R_y R_y - (\partial_x\beta_x)\beta_x B_x + (\partial_x\beta_x)\beta_x S_x + \beta_x(\partial_x\beta_x)\beta_x(\partial_x\beta_x) +$$

$$+ \beta_y(\partial_x\beta_x)\beta_x(\partial_x\beta_y) - \beta_y(\partial_x\beta_x)\beta_x R_z +$$

$$+ \beta_z(\partial_x\beta_x)\beta_x(\partial_x\beta_z) + \beta_z(\partial_x\beta_x)\beta_x R_y - (\partial_x\beta_y)\beta_y B_x + R_z\beta_y B_x$$

$$+ (\partial_x\beta_y)\beta_y S_x - R_z\beta_y S_x - (\partial_x\beta_y)R_z + R_z R_z + \beta_x(\partial_x\beta_y)\beta_y(\partial_x\beta_x) -$$

$$- \beta_x R_z\beta_y(\partial_x\beta_x) + \beta_y(\partial_x\beta_y)\beta_y(\partial_x\beta_y) - \beta_y(\partial_x\beta_y)\beta_y R_z -$$

$$- \beta_y R_z\beta_y(\partial_x\beta_y) + \beta_y R_z\beta_y R_z +$$

$$+ \beta_z(\partial_x\beta_y)\beta_y(\partial_x\beta_z) + \beta_z(\partial_x\beta_y)\beta_y R_y - \beta_z R_z\beta_y(\partial_x\beta_z) - \beta_z R_z\beta_y R_y -$$

$$- (\partial_x\beta_z)\beta_z B_x - R_y\beta_z B_x + (\partial_x\beta_z)\beta_z S_x + R_y\beta_z S_x + (\partial_x\beta_z)R_y + R_y R_y +$$

$$+ \beta_x(\partial_x\beta_z)\beta_z(\partial_x\beta_x) + \beta_x R_y\beta_z(\partial_x\beta_x) +$$

$$+\beta_y(\partial_x\beta_z)\beta_z(\partial_x\beta_y)-\beta_y(\partial_x\beta_z)\beta_z R_z+\beta_y R_y\beta_z(\partial_x\beta_y)-\beta_y R_y\beta_z R_z+$$

$$+\beta_z(\partial_x\beta_z)\beta_z(\partial_x\beta_z)+\beta_z(\partial_x\beta_z)\beta_z R_y+\beta_z R_y\beta_z(\partial_x\beta_z)+\beta_z R_y\beta_z R_y=$$

= A színes részek nullák, a maradék pedig így írható:

$$R_{11}=-\operatorname{div}(\underline{\beta}\,\partial_x\beta_x)+2\,(D_y R_y-D_z R_z)$$

A második tag így alakítható:

$$2\,(D_y R_y-D_z R_z)=$$

$$=1/2\,(\partial_z\beta_x+\partial_x\beta_z)(\partial_z\beta_x-\partial_x\beta_z)-1/2\,(\partial_x\beta_y+\partial_y\beta_x)(\partial_x\beta_y-\partial_y\beta_x)=$$

$$=1/2\,((\partial_z\beta_x)^2-(\partial_x\beta_z)^2-(\partial_x\beta_y)^2+(\partial_y\beta_x)^2)=$$

$$=1/2\,((\partial_x\beta_x)^2+(\partial_y\beta_x)^2+(\partial_z\beta_x)^2-(\partial_x\beta_x)^2-(\partial_x\beta_y)^2-(\partial_x\beta_z)^2)=$$

$$=1/2\,((\operatorname{grad}\beta_x)^2-(\partial_x\underline{\beta})^2)$$

tehát

$$R_{11}=-\operatorname{div}(\underline{\beta}\,\partial_x\beta_x)+1/2\,((\operatorname{grad}\beta_x)^2-(\partial_x\underline{\beta})^2)$$

a végeredmény.

Szimmetriaokokból

$$R_{22}=-\operatorname{div}(\underline{\beta}\,\partial_y\beta_y)+1/2\,((\operatorname{grad}\beta_y)^2-(\partial_y\underline{\beta})^2)$$

$$R_{33}=-\operatorname{div}(\underline{\beta}\,\partial_z\beta_z)+1/2\,((\operatorname{grad}\beta_z)^2-(\partial_z\underline{\beta})^2)$$

Adjuk össze a három tagot:

$R_{11}+R_{22}+R_{33}=-\operatorname{div}(\underline{\beta}\operatorname{div}\underline{\beta})$ lesz,

a többi tag összege nulla lesz!

Kaptunk tehát egy egyenletet: $\operatorname{div}(\underline{\beta}\operatorname{div}\underline{\beta})=0$.

Ezt a Kerre is ki lehet számolni.

A továbbiakhoz még R_{12}–t kell kiszámolni, utána elemezzük a kapott eredményt.

$$R_{12}=-\partial_j\Gamma_{12}{}^j+\Gamma_{1m}{}^j\Gamma_{2j}{}^m=$$

$$=-\partial_x(\beta_x D_z)-\partial_y(\beta_y D_z)-\partial_z(\beta_z D_z)+\Gamma_{1m}{}^j\Gamma_{12j}{}^m=$$

$$= -\operatorname{div}(\underline{\beta} D_z) +$$

$$+(B_x - S_x)(B_y - S_y) - D_z \beta_x (B_x - S_x) - (\partial_y \beta_y)(\beta_y (B_x - S_x) + R_z) -$$

$$-D_x(\beta_z(B_x - S_x) - R_y) -$$

$$-(\partial_x\beta_x)(\beta_x(B_y-S_y)-R_z) + \beta_x(\partial_x\beta_x)\beta_x D_z + \beta_y(\partial_x\beta_x)\beta_x(\partial_y\beta_y)+$$

$$+\beta_z(\partial_x\beta_x)\beta_x D_x -$$

$$-D_z\beta_y(B_y-S_y)+\beta_x D_z \beta_y D_z + \beta_y D_z \beta_y(\partial_y\beta_y)+\beta_z D_z\beta_y D_x -$$

$$- D_y(\beta_z(B_y-S_y)+R_x)+\beta_x D_y\beta_z D_z+\beta_y D_y \beta_z(\partial_y\beta_y)+\beta_z D_y\beta_z D_x =$$

$$= -\operatorname{div}(\underline{\beta}D_z)+$$

$$+B_xB_y - B_xS_y - S_xB_y + S_xS_y - (\partial_y\beta_x)\beta_xB_x + (\partial_y\beta_x)\beta_xS_x - R_z\beta_xB_x + R_z\beta_xS_x -$$

$$-(\partial_y\beta_y)\beta_yB_x + (\partial_y\beta_y)\beta_yS_x - (\partial_y\beta_y)R_z - (\partial_y\beta_z)\beta_zB_x + (\partial_y\beta_z)\beta_zS_x +$$

$$+(\partial_y\beta_z)R_y + R_x\beta_zB_x - R_x\beta_zS_x - R_xR_y - (\partial_x\beta_x)\beta_xB_y + (\partial_x\beta_x)\beta_xS_y +$$

$$+(\partial_x\beta_x)R_z + \beta_x(\partial_x\beta_x)\beta_x(\partial_y\beta_x) + \beta_x(\partial_x\beta_x)\beta_xR_z + \beta_y(\partial_x\beta_x)\beta_x(\partial_y\beta_y)+$$

$$+\beta_z(\partial_x\beta_x)\beta_x(\partial_y\beta_z) - \beta_z(\partial_x\beta_x)\beta_xR_x - (\partial_x\beta_y)\beta_yB_y + (\partial_x\beta_y)\beta_yS_y + R_z\beta_yB_y -$$

$$-R_z\beta_yS_y + \beta_x(\partial_y\beta_x)\beta_y(\partial_x\beta_y) + \beta_xR_z\beta_y(\partial_x\beta_y) - \beta_x(\partial_y\beta_x)\beta_yR_z - \beta_xR_z\beta_yR_z +$$

$$+\beta_y(\partial_x\beta_y)\beta_y(\partial_y\beta_y) - \beta_yR_z\beta_y(\partial_y\beta_y) + \beta_z(\partial_x\beta_y)\beta_y(\partial_y\beta_z) - \beta_zR_z\beta_y(\partial_y\beta_z) -$$

$$-\beta_z(\partial_x\beta_y)\beta_yR_x + \beta_zR_z\beta_yR_x - (\partial_x\beta_z)\beta_zB_y + (\partial_x\beta_z)\beta_zS_y - (\partial_x\beta_z)R_x +$$

$$- R_y\beta_zB_y + R_y\beta_zS_y - R_yR_x +$$

$$+\beta_x(\partial_x\beta_z)\beta_z(\partial_y\beta_x) + \beta_xR_y\beta_z(\partial_y\beta_x) + \beta_x(\partial_x\beta_z)\beta_zR_z + \beta_xR_y\beta_zR_z +$$

$$+\beta_y(\partial_x\beta_z)\beta_z(\partial_y\beta_y) + \beta_yR_y\beta_z(\partial_y\beta_y)+$$

$$+\beta_z(\partial_x\beta_z)\beta_z(\partial_y\beta_z) + \beta_zR_y\beta_z(\partial_y\beta_z) - \beta_z(\partial_x\beta_z)\beta_zR_x - \beta_zR_y\beta_zR_x$$

Két lépésben végezzük el a színezést, az áttekinthetőség kedvéért: A színessel kiemeltek nullák!

$$R_{12} = -\operatorname{div}(\underline{\beta}D_z) -$$

$$-R_z\beta_xB_x - (\partial_y\beta_y)R_z + (\partial_y\beta_z)R_y + R_x\beta_zB_x - R_xR_y - (\partial_x\beta_x)\beta_xB_y +$$

$$+(\partial_x\beta_x)R_z + \beta_x(\partial_x\beta_x)\beta_x(\partial_y\beta_x) + \beta_x(\partial_x\beta_x)\beta_xR_z + \beta_y(\partial_x\beta_x)\beta_x(\partial_y\beta_y)+$$

$$+\beta_z(\partial_x\beta_x)\beta_x(\partial_y\beta_z) - \beta_z(\partial_x\beta_x)\beta_xR_x - (\partial_x\beta_y)\beta_yB_y + R_z\beta_yB_y +$$

$$+ \beta_x (\partial_y \beta_x) \beta_y (\partial_x \beta_y) + \beta_x R_z \beta_y (\partial_x \beta_y) - \beta_x (\partial_y \beta_x) \beta_y R_z + \beta_y (\partial_x \beta_y) \beta_y (\partial_y \beta_y) -$$

$$- \beta_y R_z \beta_y (\partial_y \beta_y) + \beta_z (\partial_x \beta_y) \beta_y (\partial_y \beta_z) - \beta_z R_z \beta_y (\partial_y \beta_z) -$$

$$- \beta_z (\partial_x \beta_y) \beta_y R_x - (\partial_x \beta_z) \beta_z B_y - (\partial_x \beta_z) R_x - R_y \beta_z B_y - R_y R_x +$$

$$+ \beta_x (\partial_x \beta_z) \beta_z (\partial_y \beta_x) + \beta_x R_y \beta_z (\partial_y \beta_x) + \beta_x (\partial_x \beta_z) \beta_z R_z +$$

$$+ \beta_y (\partial_x \beta_z) \beta_z (\partial_y \beta_y) + \beta_y R_y \beta_z (\partial_y \beta_y) +$$

$$+ \beta_z (\partial_x \beta_z) \beta_z (\partial_y \beta_z) + \beta_z R_y \beta_z (\partial_y \beta_z) - \beta_z (\partial_x \beta_z) \beta_z R_x =$$

$$= - \operatorname{div} (\underline{\beta} D_z) -$$

$$- (\partial_y \beta_y) R_z + (\partial_y \beta_z) R_y - R_x R_y + (\partial_x \beta_x) R_z - (\partial_x \beta_z) R_x - R_y R_x =$$

$$= - \operatorname{div} (\underline{\beta} D_z) +$$

$$+ 1/2 (- (\partial_y \beta_y) (\partial_x \beta_y) + (\partial_y \beta_y) (\partial_y \beta_x) + (\partial_y \beta_z) (\partial_z \beta_x) -$$

$$- (\partial_y \beta_z) (\partial_x \beta_z) - (\partial_y \beta_z) (\partial_z \beta_x) + (\partial_y \beta_z) (\partial_x \beta_z) + (\partial_z \beta_y) (\partial_z \beta_x) -$$

$$- (\partial_z \beta_y) (\partial_x \beta_z) + (\partial_x \beta_x) (\partial_x \beta_y) - (\partial_x \beta_x) (\partial_y \beta_x) - (\partial_x \beta_z) (\partial_y \beta_z) +$$

$$+ (\partial_x \beta_z) (\partial_z \beta_y)) =$$

= A zöldek nullák, marad a piros és a kék:

$$R_{12} = - \operatorname{div} (\underline{\beta} D_z) - 1/2 \left((\operatorname{grad} \beta_x) (\operatorname{grad} \beta_y) - (\partial_x \underline{\beta}) (\partial_y \underline{\beta}) \right).$$

Szimmetriaokokból

$$R_{12} = - \operatorname{div} (\underline{\beta} D_z) - 1/2 \left((\operatorname{grad} \beta_x) (\operatorname{grad} \beta_y) - (\partial_x \underline{\beta}) (\partial_y \underline{\beta}) \right).$$

$$R_{23} = - \operatorname{div} (\underline{\beta} D_x) - 1/2 \left((\operatorname{grad} \beta_y) (\operatorname{grad} \beta_z) - (\partial_y \underline{\beta}) (\partial_z \underline{\beta}) \right).$$

$$R_{31} = - \operatorname{div} (\underline{\beta} D_y) - 1/2 \left((\operatorname{grad} \beta_z) (\operatorname{grad} \beta_x) - (\partial_z \underline{\beta}) (\partial_x \underline{\beta}) \right).$$

Most már elemezhetjük az eredményt.

Először az R_{00} és az R_{01}, R_{02}, R_{03} egyenleteket egyeztetjük össze:

$$R_{00} = \operatorname{divgrad} \frac{\beta^2}{2} - \operatorname{div} \left(\underline{\beta} \left(\underline{\beta} \operatorname{grad} \frac{\beta^2}{2}\right)\right) + 2 \left(B^2 - R^2 + S^2 - 2 \underline{B}\, \underline{S} \right)$$

$$\underline{R}_0 = \operatorname{rot} (\underline{\beta} \times (\underline{B} - \underline{S})) - \underline{\beta} \operatorname{div} (\underline{B} - \underline{S}) - \underline{T}.$$

Mint tudjuk, az $\underline{R}_0$ az $R_{01,}$ $R_{02,}$ R_{03} egyenleteket sűríti egy képletbe.

Most némi vektoranalitikai ismeret következik:

(V1) div ($\underline{a} \times \underline{b}$) = $\underline{b}$ rot $\underline{a}$ – $\underline{a}$ rot $\underline{b}$

(V2) rot ($\underline{a} \times \underline{b}$) = ($\underline{b}$, grad) $\underline{a}$ – ($\underline{a}$, grad) $\underline{b}$ + $\underline{a}$ div $\underline{b}$ – $\underline{b}$ div $\underline{a}$

(V3) div (f $\underline{a}$) = f div $\underline{a}$ + $\underline{a}$ grad f

(V4) rot (f $\underline{a}$) = f rot $\underline{a}$ + grad f × $\underline{a}$.

(V5) $\underline{a} \times (\underline{b} \times \underline{c}) = \underline{b}\,(\underline{a}, \underline{c}) - \underline{c}\,(\underline{a}, \underline{b})$

Eme ismeretek birtokában elkezdhetjük átalakítani az egyenleteinket.

$\mathrm{div}\,(\underline{\beta} \times (\underline{\beta} \times (\underline{B} - \underline{S}))) = (\underline{\beta} \times (\underline{B} - \underline{S}))\,\mathrm{rot}\,\underline{\beta} - \underline{\beta}\,\mathrm{rot}\,(\underline{\beta} \times (\underline{B} - \underline{S}))$ a (V1) miatt,

$\underline{\beta}\,\mathrm{rot}\,(\underline{\beta} \times (\underline{B} - \underline{S})) = \underline{\beta}\,(\ \underline{\beta}\,\mathrm{div}\,(\underline{B} - \underline{S}) + \underline{T}) = \beta^2\,\mathrm{div}\,(\underline{B} - \underline{S}) + \underline{\beta}\,\underline{T}$

mert $\underline{R}_0 = 0$, továbbá

$$\mathrm{div}\,(\underline{B} - \underline{S}) = \mathrm{div}\,\underline{B} - \mathrm{div}\,\underline{S} = \mathrm{divgrad}\,\frac{\beta^2}{2} - \mathrm{div}\,\underline{S}.$$

$\mathrm{div}\,\underline{S} = \mathrm{div}\,(\underline{\beta} \times \underline{R}) = \underline{R}\,\mathrm{rot}\,\underline{\beta} - \underline{\beta}\,\mathrm{rot}\,\underline{R} = 2\,R^2 - \underline{\beta}\,\underline{T}$ a (V1) miatt, végül

$$\underline{\beta} \times (\underline{\beta} \times (\underline{B} - \underline{S})) = \underline{\beta}\,(\underline{\beta}\,(\underline{B} - \underline{S})) - \beta^2\,(\underline{B} - \underline{S}).$$

A vektorok vegyesszorzási szabálya is kell:

(V5) $\underline{a}\,(\underline{b} \times \underline{c}) = \underline{c}\,(\underline{a} \times \underline{b}) = \underline{b}\,(\underline{c} \times \underline{a})$.

$$\mathrm{div}\,(\underline{\beta} \times (\underline{\beta} \times (\underline{B} - \underline{S}))) = (\underline{\beta} \times (\underline{B} - \underline{S}))\,2\,\underline{R} - \beta^2\,\mathrm{div}\,(\underline{B} - \underline{S}) - \underline{\beta}\,\underline{T} =$$

$$= \mathrm{div}\,(\underline{\beta}\,(\underline{\beta}\,(\underline{B} - \underline{S})) - \beta^2\,(\underline{B} - \underline{S})) = \mathrm{div}\,(\underline{\beta}\,(\underline{\beta}\,\mathrm{grad}\,\frac{\beta^2}{2})) - \mathrm{div}\,(\underline{\beta}\,(\underline{\beta}\,\underline{S})) -$$

$$- \mathrm{div}\,(\beta^2\,\underline{B}) + \mathrm{div}\,(\beta^2\,\underline{S}).$$

A második tag nulla mert ($\underline{\beta}\,\underline{S}$) = 0.

$$\mathrm{div}\,(\beta^2\,\underline{B}) = \beta^2\,\mathrm{div}\,\underline{B} + \underline{B}\,\mathrm{grad}\,\beta^2 = \beta^2\,\mathrm{divgrad}\,\frac{\beta^2}{2} + 2\,B^2$$

mert B= grad $\frac{\beta^2}{2}$.

$$\text{div}\,(\beta^2\,\underline{S}) = \beta^2\,\text{div}\,\underline{S} + \underline{S}\,\text{grad}\,\beta^2 = \beta^2\,\text{div}\,\underline{S} + 2\,\underline{B}\,\underline{S}.$$

$$\text{div}\,(\underline{\beta}\,(\underline{\beta}\,\text{grad}\,\frac{\beta^2}{2})) = \text{divgrad}\,\frac{\beta^2}{2} + 2\,(\,B^2 - R^2 + S^2 - 2\,\underline{B}\,\underline{S})$$

mert R_{00} =0.

A fentieket egybevetve:

$$(\underline{\beta}\times(\underline{B}-\underline{S}))2\underline{R} - \beta^2\,\text{div}\,(\underline{B}-\underline{S}) - \underline{\beta}\,\underline{T} = \text{divgrad}\,\frac{\beta^2}{2} + 2\,(\,B^2 - R^2 + S^2 - 2\,\underline{B}\,\underline{S}) -$$

$$-\beta^2\,\text{divgrad}\,\frac{\beta^2}{2} - 2\,B^2 + \beta^2\,\text{div}\,\underline{S} + 2\,\underline{B}\,\underline{S}.$$

A baloldal továbbalakítható:

$$2\underline{R}\,(\underline{\beta}\times\underline{B}) - 2\underline{R}\,(\underline{\beta}\times\underline{S}) = -2\underline{B}\,(\underline{\beta}\times\underline{R}) + 2\underline{S}\,(\underline{\beta}\times\underline{R})\text{ a (V6) miatt, } = 2S^2 - 2\underline{B}\,\underline{S}$$

$$-\beta^2\,\text{div}\,(\underline{B}-\underline{S}) = -\beta^2\,\text{divgrad}\,\frac{\beta^2}{2} + \beta^2\,\text{div}\,\underline{S}.$$

Ezekkel az egyenlet:

$$2S^2 - 2\underline{B}\,\underline{S} - \beta^2\,\text{divgrad}\,\frac{\beta^2}{2} + \beta^2\,\text{div}\,\underline{S} - \underline{\beta}\,\underline{T} =$$

$$= \text{divgrad}\,\frac{\beta^2}{2} + 2\,(\,B^2 - R^2 + S^2 - 2\,\underline{B}\,\underline{S}) -$$

$$-\beta^2\,\text{divgrad}\,\frac{\beta^2}{2} - 2\,B^2 + \beta^2\,\text{div}\,\underline{S} + 2\,\underline{B}\,\underline{S}.$$

A színesek kiesnek!   Maradt:

$$\text{divgrad}\,\frac{\beta^2}{2} - 2R^2 + \underline{\beta}\,\underline{T} = 0,$$

azaz

$$\text{divgrad}\,\frac{\beta^2}{2} - \text{div}\,S = 0.$$

Látni fogjuk, hogy külön divgrad $\frac{\beta^2}{2}$=0 és külön div $\underline{S}$ =0 is teljesül, akkor pedig

div (grad $\frac{\beta^2}{2}$ - 2 $\underline{S}$) =0 is is igaz, vagyis végül is div $\underline{a}$ =0 !!

$\underline{a}$ = gyorsulás = $\partial v/\partial t + (\underline{v}, \text{grad})\underline{v} = \text{grad}\ \frac{v^2}{2} - \underline{v} \times \text{rot}\ \underline{v}$ mert stacionáris.

Tehát beláttuk hogy div $\underline{a}$ =0. Ezért az egyenletért tettük meg ezt a nagy utat!

Most pedig az utolsó felvonás következik! Megmutatjuk, hogy az R_{01}, R_{02}, R_{03}, R_{11}, R_{12}, R_{13}, R_{22}, R_{23}, R_{33} egyenletekből le lehet vezetni a rot $\underline{\beta}$ = 0 feltételünket! Ehhez az R_{01}, R_{02}, R_{03} egyenletek eredeti alakját használjuk: R_{ik} =0 értelmében mindegyik kifejezés értéke nulla!

$R_{01} = -\text{div}\,(\underline{\beta}\,(B_x - S_x)) - T_x + (\underline{B} - \underline{S})\ \text{grad}\ \beta_x$

$R_{02} = -\text{div}\,(\underline{\beta}\,(B_y - S_y)) - T_y + (\underline{B} - \underline{S})\ \text{grad}\ \beta_y$

$R_{03} = -\text{div}\,(\underline{\beta}\,(B_z - S_z)) - T_z + (\underline{B} - \underline{S})\ \text{grad}\ \beta_z$

$R_{11} = -\text{div}\,(\underline{\beta}\,\partial_x \beta_{x)} + 1/2\ ((\text{grad}\ \beta_{x)}{}^2 - (\partial_x \underline{\beta}_)\,{}^{2)}$ a végeredmény.

$R_{22} = -\text{div}\,(\underline{\beta}\,\partial_y \beta_{y)} + 1/2\ ((\text{grad}\ \beta_{y)}{}^2 - (\partial_y \underline{\beta}_)\,{}^{2)}$

$R_{33} = -\text{div}\,(\underline{\beta}\,\partial_z \beta_{z)} + 1/2\ ((\text{grad}\ \beta_{z)}{}^2 - (\partial_z \underline{\beta}_)\,{}^{2)}$

$R_{12} = -\text{div}\,(\underline{\beta}\,D_{z)} - 1/2\ (\ (\text{grad}\ \beta_x)\ (\text{grad}\ \beta_y) - (\partial_x \underline{\beta}_)\ (\partial_y \underline{\beta}_)).$

$R_{23} = -\text{div}\,(\underline{\beta}\,D_{x)} - 1/2\ (\ (\text{grad}\ \beta_y)\ (\text{grad}\ \beta_z) - (\partial_y \underline{\beta}_)\ (\partial_z \underline{\beta}_)).$

$R_{31} = -\text{div}\,(\underline{\beta}\,D_{y)} - 1/2\ (\ (\text{grad}\ \beta_z)\ (\text{grad}\ \beta_x) - (\partial_z \underline{\beta}_)\ (\partial_x \underline{\beta}_)).$

Az átalakítást R_{01} esetében mutatom meg, a többi hasonlóan megy.

$R_{01} = -\text{div}\,(\underline{\beta}\,(B_x - S_x)) - T_x + (\underline{B} - \underline{S})\ \text{grad}\ \beta_x$

$\text{div}\,(\underline{\beta}\,(B_x - S_x)) = \text{div}\,(\underline{\beta}\,(\beta_x\partial_x \beta_x + \beta_y\partial_x \beta_y + \beta_z\partial_x \beta_z - \beta_y R_z + \beta_z R_y)) =$

$= \text{div}\,(\underline{\beta}\,(\beta_x\partial_x \beta_x + \beta_y D_z + \beta_z D_y)) =$

$= \text{div}\,(\underline{\beta}\,(\beta_x\partial_x \beta_{x)}) + \text{div}\,(\underline{\beta}\,(\beta_y D_{z)}) + \text{div}\,(\underline{\beta}\,(\beta_z D_y)) =$

$= \beta_x\,\text{div}\,(\underline{\beta}\,(\partial_x \beta_{x)}) + \underline{\beta}\,(\partial_x \beta_{x)}\text{grad}\ \beta_x + \beta_y\,\text{div}\,(\underline{\beta}\,D_z) + \underline{\beta}\,D_z\,\text{grad}\ \beta_y +$

$+ \beta_z\,\text{div}\,(\underline{\beta}\,D_y) + \underline{\beta}\,D_y\,\text{grad}\ \beta_z =$ most alkalmazzuk az R_{11}, R_{12} és R_{31} képleteket:

$=1/2\ \{\beta_x\,((\text{grad}\ \beta_{x)}{}^2 - (\partial_x \underline{\beta}_)\,{}^{2)} + \beta_y\,((\text{grad}\ \beta_x)\ (\text{grad}\ \beta_y) - (\partial_x \underline{\beta}_)\ (\partial_y \underline{\beta}_)) +$

$$+ \beta_z\ ((\mathrm{grad}\ \beta_z)\ (\mathrm{grad}\ \beta_x) - (\partial_z \underline{\beta})\ (\partial_x \underline{\beta}))\ \} + \underline{\beta}\ (\partial_x \beta_x) \mathrm{grad}\ \beta_x + \underline{\beta}\ D_z\ \mathrm{grad}\ \beta_y +$$

$$+ \underline{\beta}\ D_y\ \mathrm{grad}\ \beta_z =$$

$$= 1/2\ \{\beta_x ((\mathrm{grad}\ \beta_x)^2 - (\partial_x \underline{\beta})^2) + \beta_y ((\mathrm{grad}\ \beta_x)\ (\mathrm{grad}\ \beta_y) - (\partial_x \underline{\beta})\ (\partial_y \underline{\beta})) +$$

$$+ \beta_z\ ((\mathrm{grad}\ \beta_z)\ (\mathrm{grad}\ \beta_x) - (\partial_z \underline{\beta})\ (\partial_x \underline{\beta}))\ \} + \underline{\beta}\ \mathrm{grad}\ \beta_x (\partial_x \beta_x) +$$

$$+ \underline{\beta}\ \mathrm{grad}\ \beta_y (\partial_y \beta_x + R_z) + \underline{\beta}\ \mathrm{grad}\ \beta_z (\partial_z \beta_x - R_y).$$

$$(\underline{B} - \underline{S})\ \mathrm{grad}\ \ \beta_x =$$

$$= (\beta_x (\partial_x \beta_x) + \beta_y (\partial_x \beta_y) + \beta_z (\partial_x \beta_z) - 1/2\beta_y (\partial_x \beta_y) + 1/2\beta_y (\partial_y \beta_x) +$$

$$+ 1/2\beta_z (\partial_z \beta_x) - 1/2\beta_z (\partial_x \beta_z)))\ (\partial_x \beta_x) +$$

$$+ (\beta_x (\partial_y \beta_x) + \beta_y (\partial_y \beta_y) + \beta_z (\partial_y \beta_z) - 1/2\beta_z (\partial_y \beta_z) + 1/2\beta_z (\partial_z \beta_y) +$$

$$+ 1/2\beta_x (\partial_x \beta_y) - 1/2\beta_x (\partial_y \beta_x))\ (\partial_y \beta_x) +$$

$$+ (\beta_x (\partial_z \beta_x) + \beta_y (\partial_z \beta_y) + \beta_z (\partial_z \beta_z) - 1/2\beta_x (\partial_z \beta_x) + 1/2\beta_x (\partial_x \beta_z) +$$

$$+ 1/2\beta_y (\partial_y \beta_z) - 1/2\beta_y (\partial_z \beta_y))\ (\partial_z \beta_x)$$

a piros és narancs részek összevonhatók, a feketék pedig

$1/2\beta_x (\partial_x \beta_z) = \beta_x (\partial_x \beta_z) - 1/2\beta_x (\partial_x \beta_z)$ módon kettévehetők:

$$(\underline{B} - \underline{S})\ \mathrm{grad}\ \ \beta_x = (\beta_x (\partial_x \beta_x) + \beta_y (\partial_y \beta_x) + \beta_z (\partial_z \beta_x))\ (\partial_x \beta_x) +$$

$$+ (\beta_y (\partial_y \beta_y) + \beta_z (\partial_z \beta_y) + \beta_x (\partial_x \beta_y))\ (\partial_y \beta_x) +$$

$$+ (\beta_z (\partial_z \beta_z) + \beta_x (\partial_x \beta_z) + \beta_y (\partial_y \beta_z))\ (\partial_z \beta_x) +$$

$$+ 1/2\ \{\beta_y (\partial_x \beta_y)(\partial_x \beta_x) + \beta_z (\partial_x \beta_z)(\partial_x \beta_x) - \beta_y (\partial_y \beta_x)(\partial_x \beta_x) - \beta_z (\partial_z \beta_x)(\partial_x \beta_x) +$$

$$+ \beta_z (\partial_y \beta_z)(\partial_y \beta_x) + \beta_x (\partial_y \beta_x)(\partial_y \beta_x) - \beta_z (\partial_z \beta_y)(\partial_y \beta_x) - \beta_x (\partial_x \beta_y)(\partial_y \beta_x) +$$

$$+ \beta_x (\partial_z \beta_x)(\partial_z \beta_x) + \beta_y (\partial_z \beta_y)(\partial_z \beta_x) - \beta_x (\partial_x \beta_z)(\partial_z \beta_x) - \beta_y (\partial_y \beta_z)(\partial_z \beta_x)\} =$$

$$= \underline{\beta}\ \mathrm{grad}\ \beta_x (\partial_x \beta_x) + \underline{\beta}\ \mathrm{grad}\ \beta_y (\partial_y \beta_x) + \underline{\beta}\ \mathrm{grad}\ \beta_z (\partial_z \beta_x) +$$

$$+ 1/2\ \{\beta_y (\partial_x \beta_y)(\partial_x \beta_x) + \beta_z (\partial_x \beta_z)(\partial_x \beta_x) - \beta_y (\partial_y \beta_x)(\partial_x \beta_x) - \beta_z (\partial_z \beta_x)(\partial_x \beta_x) +$$

$$+ \beta_z (\partial_y \beta_z)(\partial_y \beta_x) + \beta_x (\partial_y \beta_x)(\partial_y \beta_x) - \beta_z (\partial_z \beta_y)(\partial_y \beta_x) - \beta_x (\partial_x \beta_y)(\partial_y \beta_x) +$$

$$+ \beta_x (\partial_z \beta_x)(\partial_z \beta_x) + \beta_y (\partial_z \beta_y)(\partial_z \beta_x) - \beta_x (\partial_x \beta_z)(\partial_z \beta_x) - \beta_y (\partial_y \beta_z)(\partial_z \beta_x)\}$$

A sötétkék és a zöld kifejezés akkor egyenlő, ha

$$1/2\ \{\beta_x((\text{grad}\,\beta_x)^2 - (\partial_x \underline{\beta})^{2)} + \beta_y((\text{grad}\,\beta_x)(\text{grad}\,\beta_y) - (\partial_x \underline{\beta})(\partial_y \underline{\beta})) +$$

$$+ \beta_z\ ((\text{grad}\,\beta_z)(\text{grad}\,\beta_x) - (\partial_z \underline{\beta})(\partial_x \underline{\beta}))\ \} + \underline{\beta}\ \text{grad}\,\beta_y R_z - \underline{\beta}\ \text{grad}\,\beta_z\ R_y =$$

$$=1/2\ \{\beta_y(\partial_x \beta_y)(\partial_x \beta_x) + \beta_z(\partial_x \beta_z)(\partial_x \beta_x) - \beta_y(\partial_y \beta_x)(\partial_x \beta_x) - \beta_z(\partial_z \beta_x)(\partial_x \beta_x) +$$

$$+ \beta_z(\partial_y \beta_z)(\partial_y \beta_x) + \beta_x(\partial_y \beta_x)(\partial_y \beta_x) - \beta_z(\partial_z \beta_y)(\partial_y \beta_x) - \beta_x(\partial_x \beta_y)(\partial_y \beta_x) +$$

$$+ \beta_x(\partial_z \beta_x)(\partial_z \beta_x) + \beta_y(\partial_z \beta_y)(\partial_z \beta_x) - \beta_x(\partial_x \beta_z)(\partial_z \beta_x) - \beta_y(\partial_y \beta_z)(\partial_z \beta_x)\}$$

Tehát: (szorozzuk mindkét oldalt kettővel)

$$\beta_x(\partial_x \beta_x)(\partial_x \beta_x) + \beta_x(\partial_y \beta_x)(\partial_y \beta_x) + \beta_x(\partial_z \beta_x)(\partial_z \beta_x) - \beta_x(\partial_x \beta_x)(\partial_x \beta_x) -$$

$$- \beta_x(\partial_x \beta_y)(\partial_x \beta_y) - \beta_x(\partial_x \beta_z)(\partial_x \beta_z) +$$

$$+ \beta_y(\partial_x \beta_x)(\partial_x \beta_y) + \beta_y(\partial_y \beta_x)(\partial_y \beta_y) + \beta_y(\partial_z \beta_x)(\partial_z \beta_y) - \beta_y(\partial_x \beta_x)(\partial_y \beta_x) -$$

$$- \beta_y(\partial_x \beta_y)(\partial_y \beta_y) - \beta_y(\partial_x \beta_z)(\partial_y \beta_z) +$$

$$+\beta_z(\partial_x \beta_x)(\partial_x \beta_z) + \beta_z(\partial_y \beta_x)(\partial_y \beta_z) + \beta_z(\partial_z \beta_x)(\partial_z \beta_z) - \beta_z(\partial_x \beta_x)(\partial_z \beta_x) -$$

$$- \beta_z(\partial_x \beta_y)(\partial_z \beta_y) - \beta_z(\partial_x \beta_z)(\partial_z \beta_z) -$$

$$+ (\beta_x(\partial_x \beta_y) + \beta_y(\partial_y \beta_y) + \beta_z(\partial_z \beta_y))((\partial_x \beta_y) - (\partial_y \beta_x)) +$$

$$- (\beta_x(\partial_x \beta_z) + \beta_y(\partial_y \beta_z) + \beta_z(\partial_z \beta_z))((\partial_z \beta_x) - (\partial_x \beta_z)) -$$

$$- \{\beta_y(\partial_x \beta_y)(\partial_x \beta_x) + \beta_z(\partial_x \beta_z)(\partial_x \beta_x) - \beta_y(\partial_y \beta_x)(\partial_x \beta_x) - \beta_z(\partial_z \beta_x)(\partial_x \beta_x) +$$

$$+ \beta_z(\partial_y \beta_z)(\partial_y \beta_x) + \beta_x(\partial_y \beta_x)(\partial_y \beta_x) - \beta_z(\partial_z \beta_y)(\partial_y \beta_x) - \beta_x(\partial_x \beta_y)(\partial_y \beta_x) +$$

$$+ \beta_x(\partial_z \beta_x)(\partial_z \beta_x) + \beta_y(\partial_z \beta_y)(\partial_z \beta_x) - \beta_x(\partial_x \beta_z)(\partial_z \beta_x) - \beta_y(\partial_y \beta_z)(\partial_z \beta_x\ \} = 0$$

A színek szerinti egyeztetés stimmel, tehát ez a kifejezés valóban nulla!

Tehát div $(\underline{\beta}\,(B_x - S_x)) = (\underline{B} - \underline{S})$ grad $\beta_{x,}$

és akkor

$$R_{01} = -\text{div}\,(\underline{\beta}\,(B_x - S_x)) - T_x\ \ + (\underline{B} - \underline{S})\ \text{grad}\ \beta_x = - T_x$$

mindössze, és akkor $R_{01} = 0$ – ból következik $T_x = 0$, azaz végül is $\underline{T} = 0$!

A div $\underline{S} = 0$ – ból $2\ R^2 = \underline{\beta}\ \underline{T}$ adódik, és mivel $\underline{T} = 0$, így végül is $\underline{R} = 0$ azaz rot $\underline{\beta} = 0$!

Ezzel utunk végére értünk. Beláttuk hogy az általános esetben is rot $\underline{\beta} = 0$ teljesül. Nem kell külön feltételként kikötni. Az egyetlen probléma az, hogy ezt az egyenletet még nem sikerült a Kerr - metrikára igazolni. Ha majd sikerül, az lesz a végső igazolása teóriánknak.

A későbbiekben azt fogom leírni, hogyan adódik ki egyszerűen a kvantumfizika a rugalmas éter elméletéből, és hogyan lehet összekapcsolni a Hamilton formalizmust az áramló éter modelljével, a Hangterjedéssel Áramló Közegben, amelynek leíró formalizmusa hasonló lesz az Általános Relativitáselmélet eszközeihez, itt is kovariáns egyenleteket kapunk.

Elemezni kell még a nemstacionáris esetet is, és azt, amikor a T_{ik} se nulla.

Végül a szuperfolyékony éter hidrodinamikai modelljével igazolni kell azt is, hogy az éter tényleg így viselkedik, valóban a rot $\underline{\beta} = 0$ és a divgrad $\frac{\beta^2}{2}=0$ egyenletek írják le.

Ha mindez megvan, akkor mondhatjuk hogy megvan az éter konzisztens elmélete. Remélem, még az idén elkészülök vele. És akkor semmi akadálya, hogy hivatalosan is elismerjék az éter létét. 2004.7.25